

KSCV Workshop #20

Titles and Abstracts

February 20, 2013

Adachi, Masanori (Nagoya U, Japan; m08002z@math.nagoya-u.ac.jp) *On the ampleness of positive line bundles over Levi flat manifolds*

Abstract. We give an example of a compact Levi-flat CR 3-manifold with a positive-along-leaves CR line bundle which is not ample with respect to transversely infinitely differentiable CR sections. This example shows that we cannot improve the regularity of Kodaira type embedding theorem for compact Levi-flat CR manifolds obtained by Ohsawa and Sibony.

Reference: [arXiv:1301.5957](https://arxiv.org/abs/1301.5957)

Ahn, Taeyong (The SRC-GAIA, triumph@postech.ac.kr) *Equidistribution in higher codimension for holomorphic endomorphisms of \mathbb{P}^k*

Abstract. In this talk, we discuss equidistribution phenomena. As a tool, we study super-potentials introduced by T. -C. Dinh and N. Sibony in [1] and present some difficulties in higher codimensional cases. Then, using Lojasiewicz inequality, we briefly show a new result on equidistribution in higher codimensional cases.

Reference

[1] Dinh, Tien-Cuong and Sibony, Nessim, "Super-potentials of positive closed currents, intersection theory and dynamics" Acta Mathematica September 2009, Volume 203, Issue 1, pp 1-82.

Cho, Hong Rae (Pusan National Univ, chohr@pusan.ac.kr) *Estimates for $\bar{\partial}$ and Hankel operators on generalized Fock space on \mathbb{C}^n*

Abstract. Let $\varphi : \mathbb{C}^n \rightarrow \mathbb{R}$ be a C^2 plurisubharmonic function on \mathbb{C}^n . Suppose that there exist $C_1, C_2 > 0$ such that $\sup_{\mathbb{C}^n} |\bar{\partial}\partial\varphi| < C_1$ and $H_\varphi(\xi, \xi)(z) \geq C_2|\xi|^2$ for $\xi \in \mathbb{R}^{2n}$ and $z \in \mathbb{C}^n$, where $H_\varphi(\xi, \xi)(z)$ is the real Hessian of φ at z . We prove $L^{p,\varphi}$ estimates for $\bar{\partial}$ on \mathbb{C}^n for all $p \in [1, \infty]$. Moreover, by using the estimates for $\bar{\partial}$, we characterize boundedness and compactness of Hankel operators with anti-holomorphic symbols on generalized Fock spaces on \mathbb{C}^n .

Deng, Fusheng (U of Chinese Academy of Sciences: fshdeng@amss.ac.cn) *Variation of volumes of pseudoconvex manifolds and the minimum principle for p.s.h. functions*

Abstract. In this talk we'll discuss various versions of the minimum principle for p.s.h functions. We interpret Berndtsson's form of the minimum principle as a result about variation of volumes of pseudoconvex domains, and then generalize it to general pseudoconvex manifolds. We'll explain a further generalization to the context of categorical quotient of Stein manifolds, where the minimum principle can be more clearly described and understood. We also want to explain Berndtsson's idea on relating the minimum principle to positivity of certain holomorphic vector bundles. Finally, we'll propose a conjecture on positivity of certain vector bundles associated to holomorphic fibration and group representation.

This talk is based on a joint work with Xiangyu Zhou.

Fornaess, John Erik (NTNU, Norway: john.fornass@math.ntnu.no) *Exposing Boundary Points*

Abstract. This is joint work with Klas Diederich and Erlend Fornaess Wold. It is motivated by a problem posed by Fusheng DENG, Q. GUAN and L. ZHANG. It concerns the problem of mapping a domain into a convex domain in such a way that a prescribed boundary point is mapped to the boundary of the convex domain.

Han, Chong Kyu (Seoul National Univ: ckhan@snu.ac.kr) *Invariant submanifolds for real vector fields of constant rank and their complex analogues*

Abstract. Given a system of vector fields on a smooth manifold that spans a plane field of constant rank, we present a systematic method and an algorithm to find submanifolds that are invariant under the flows of the vector fields. We present examples of partition into invariant submanifolds, which further gives partition into orbits. We discuss some complex analogues of the results. This is a joint work with Heungju Ahn.

Kim, Hyeseon (The SRC-GAIA: hyeseon@postech.ac.kr) *The variation formulae of Robin function on certain unbounded domains in \mathbb{C}^n*

Abstract. The Robin function of a given domain is defined through the Dirichlet problem of a domain in Riemannian manifold. One of the main reasons for the interests on Robin function is that, even if it came from the metric geometry, it gives rise to a plurisubharmonic exhaustion to bounded strongly pseudoconvex domains. The unusual feature follows by the well-known second variation formula of the Robin function (in case the domain is in a homogeneous space as a Euclidean space or a complex Lie group) in the context of variation of bounded domains.

On the other hand, if the domain is unbounded, very little analysis has been performed to this direction of study. In conjunction with the current interest on the study of unbounded (strongly) pseudoconvex domains, we shall establish the first and the second variation formulae of the Robin function for certain unbounded domains in \mathbb{C}^n . This talk is based on the collaboration with Kang-Tae Kim.

References

- [1] D. Borah and K. Verma, *Remarks on the metric induced by the Robin function*, Indiana Univ. Math. J. (3) **60** (2011), 751–802.
- [2] J.-C. Joo, *On the Levenberg-Yamaguchi formula for the Robin function*, Complex Var. Elliptic Equ. (3-4) **54** (2009), 345–353.
- [3] K.-T. Kim, N. Levenberg and H. Yamaguchi, *Robin functions for complex manifolds and applications*, Mem. Amer. Math. Soc. (984) **209** (2011), viii+111.
- [4] H. Yamaguchi, *Variations of pseudoconvex domains over \mathbb{C}^n* , Michigan Math. J. (3) **36** (1989), 415–457.

Kossovskiy, Ilya (Univ of Western Ontario: ikossovs@uwo.ca) *Analytic theory of differential equations and spherical real hypersurfaces.*

Abstract. We establish an injective correspondence between nonminimal spherical real hypersurfaces in complex affine 2-space and certain second order singular ODEs in complex plane. Using the correspondence, we give a necessary and sufficient condition for a mapping of a nonminimal hypersurface into a sphere to extend holomorphically to the complex locus. As an application, we prove the estimate $\dim \text{Aut}(M, p) \leq 5$ for the automorphism group of an arbitrary germ of a nonminimal hypersurface M in complex affine 2-space.

Nguyen, Quang Dieu (Hanoi Univ of Education: dieu.vn@yahoo.com) *Rapid approximation of holomorphic functions and pluripolar hulls*

Abstract. We say that a sequence r_m of rational functions of degree m is convergent rapidly pointwise to a function f defined on a domain D in C^n if $\lim_{m \rightarrow \infty} |r_m(z) - f(z)|^{1/m} = 0$ for every $z \in D$. The main result of my talk shows that pointwise convergence in a small subste of D implies pointwise convergence almost every where on D . By a standard complexification techniques, we obtain an analogue result for real rational functions.

Ninh, Van Thu (The SRC-GAIA: thunv@postech.ac.kr) *On the automorphism groups of models in \mathbb{C}^2*

Abstract. In this talk we consider a model

$$M_H = \{(z_1, z_2) \in \mathbb{C}^2 : \text{Re}z_2 + H(z_1) < 0\},$$

where H is a homogeneous subharmonic polynomial of degree $2m$ ($m \geq 1$) without harmonic terms. We first prove the following theorem.

Theorem 1. *Let Ω be a domain in \mathbb{C}^2 and let $p \in \partial\Omega$. Suppose that Ω satisfies Condition (M) at p and there exist a sequence $\{f_n\} \subset \text{Aut}(\Omega)$ and $q \in \Omega$ such that $\{f_n(q)\}$ converges tangentially to order $\leq 2m$ ($= \deg(H)$) to p . Then Ω is biholomorphically equivalent to the model M_H .*

Then we give an explicit description for automorphism groups of models in \mathbb{C}^2 as follows.

Theorem 2. *If $m \geq 2$, then*

(i) $\text{Aut}(\Omega_m)$ is generated by

$$\{T_t^1, T_t^2, R_\theta, S_\lambda \mid t \in \mathbb{R}, \lambda > 0, \text{ and } \exp(i\theta) \text{ is an } L\text{-root of unity}\};$$

(ii) For any generic model M_H , $\text{Aut}(M_H)$ is generated by

$$\{T_t^2, R_\theta, S_\lambda \mid t \in \mathbb{R}, \lambda > 0, \text{ and } \exp(i\theta) \text{ is an } L\text{-root of unity}\},$$

where $T_t^1, T_t^2, R_\theta, S_\lambda$ given by

$$T_t^1 : (z_1, z_2) \mapsto (z_1 + it, z_2); \quad (1)$$

$$T_t^2 : (z_1, z_2) \mapsto (z_1, z_2 + it); \quad (2)$$

$$R_\theta : (z_1, z_2) \mapsto (e^{i\theta} z_1, z_2); \quad (3)$$

$$S_\lambda : (z_1, z_2) \mapsto (\lambda z_1, \lambda^{2m} z_2), \quad (4)$$

where $t \in \mathbb{R}$, $\lambda > 0$, and $\exp(i\theta)$ is an L -root of unity.

References

- [1] E. Bedford and S. Pinchuk, *Domains in \mathbb{C}^2 with noncompact automorphism groups*, Indiana Univ. Math. Journal 47 (1998), 199–222.
- [2] F. Berteloot, *Characterization of models in \mathbb{C}^2 by their automorphism groups*, Internat. J. Math. 5 (1994), 619–634.
- [3] J. Byun and H. R. Cho, *Explicit description for the automorphism group of the Kohn-Nirenberg domain*, Math. Z. 263 (2009), no. 2, 295–305.
- [4] R. Greene, K. T. Kim and S. Krantz, *The geometry of complex domains*, Progress in Mathematics, 291. Birkhuser Boston, Inc., Boston, MA, 2011.
- [5] A. Isaev and S. Krantz, *Domains with non-compact automorphism group : A survey*, Adv. Math. 146 (1999), 1–38.
- [6] M. Kolar, *Normal forms for hypersurfaces of finite type in \mathbb{C}^2* , Math. Res. Lett. 12 (2005), 897–910.
- [7] K. Verma, *A characterization of domains in \mathbb{C}^2 with noncompact automorphism group*, Math. Ann. 334 (2009), no. 3-4, 645–701.

Park, Inyoung (PARC, Seoul National Univ: in02z@korea.ac.kr) *Compact differences of composition operators on the Bergman space over the ball*

Abstract. The compact differences of composition operators acting on the weighted L^2 -Bergman space over the unit disk is characterized by the angular derivative cancellation property and due to Moorhouse. In this talk, we present an extension of Moorhouses characterization, as well as some related results, to the ball and, at the same time, to the weighted L^p -Bergman space for the full range of p .

Peters, Han (Univ Amsterdam, Netherlands: h.peters@uva.nl) *Fatou Components in two complex dimensions.*

Abstract. The iterative behavior of rational maps in two complex variables is not nearly as well understood as in one variable. In this talk we will focus on components of the Fatou set, that is, the set where the dynamics behaves orderly. In one dimension these Fatou components have been precisely described. In higher dimensions there are many open questions, both in terms of the topology and complex structure of the components and in terms of the behavior of the orbits. I will discuss what is currently known in the literature, and present recent results with Lyubich and with Boc-Thaler and Fornæss.

Tran, Vu Khanh (Tan Tao University, Vietnam : khanh.tran@ttu.edu.vn) *Consequences of the f -property*

Abstract. Let Ω be a pseudoconvex domain in \mathbb{C}^n and z_0 be a boundary point. For a smooth monotonic increasing function $f : [1, +1) \rightarrow [1, +1)$ with $f(t) \leq t^{\frac{1}{2}}$, we say that Ω has the f -Property at z_0 if there exist a neighborhood U of z_0 , constants $c_1, c_2 > 0$ and a family of functions $\{\phi_\delta\}$ such that

- (i) ϕ_δ are plurisubharmonic and C^2 on U and $1 \leq \phi_\delta \leq 0$;
- (ii) $\partial\bar{\partial}\phi_\delta \geq c_1 f(\delta^1)^2 Id$ and $|D\phi_\delta| \leq c_2 \delta^{-1}$ for any $z \in U \cap \{z \in \Omega : \delta < r(z) < 0\}$,

where r is a defining function of Ω . We discuss the consequences of the f -Property:

- (1) Estimates and regularity of the $\bar{\partial}$ -Neumann problem;
- (2) Existence of families of bumping functions and plurisubharmonic peak functions;
- (3) Lower bounds of the Bergman metric and Kobayashi metric;
- (4) Estimates for the “type” of the boundary.

Yamaguchi, Hiroshi (Nara Women's University, Japan) *Robin function and Robin metric in higher dimension*

Abstract. (1) We first recall the classic Robin constant λ of a domain D in \mathbb{C}_z with smooth ∂D , and show the variation formula of the Robin constant $\lambda(t)$ of the moving domain $D(t)$ in \mathbb{C} with complex parameter t . Then I show its applications in the several complex variables. (2) We next introduce the Robin constant λ in the domain D in \mathbb{C}^n ($n \geq 2$). We show the variation formula of the Robin constant $\lambda(t)$ of the moving domain $D(t)$ in \mathbb{C}^n with parameter t . As application we introduce the Robin metric ds^2 in a pseudoconvex domain D in \mathbb{C}^n with smooth ∂D .

(3) We extend (2) in the complex Kähler manifold M . We establish the variation formula of the c -Robin constant $\lambda(t)$ in the moving domain $D(t)$ in M . As application we show the characterization of the pseudoconvex domain D in the homogeneous space M with smooth ∂D which is not Stein.

(4) We determine all non-Stein pseudoconvex domains D in the Hopf surface $\mathcal{H}(a; b)$ with smooth ∂D .

Yamamori, Atsushi (The SRC-GAIA: ats.yamamori@gmail.com) *On the Forelli-Rudin construction and its applications*

Abstract. For the Hartogs domain, it is known that the Bergman kernel is expressed in terms of a series involving weighted Bergman kernels of its base domain, which is called the Forelli-Rudin construction [1]. We will explain that the Forelli-Rudin construction plays an important role in establishing explicit formulas of the Bergman kernels of some Hartogs domains. If the time permits, we will discuss deflation type identities for a certain class of domains by using a generalized Forelli-Rudin construction. This talk is based on the papers [2, 3, 4] listed below

References

- [1] E. Ligocka, Forelli-Rudin Constructions and weighted Bergman projections, *Studia Math.*, 94:257–272, 1989.
- [2] A. Yamamori, The Bergman kernel of the Fock-Bargmann-Hartogs domain and the polylogarithm function, *Complex Variables Elliptic Eqs.* (To appear), doi:10.1080/17476933.2011.620098.
- [3] A. Yamamori, A remark on the Bergman kernels of the Cartan-Hartogs domains, *C. R. Acad. Sci. Paris, Ser. I* 350 (2012),157–160.
- [4] A. Yamamori, A note on the Bergman kernel of a certain Hartogs domain, *C. R. Acad. Sci. Paris, Ser. I* 350 (2012),827–829.

Zhou, Xiang-Yu (Chinese Academy of Sciences: xyzhou@math.ac.cn) *Optimal constant in L^2 extension and a conjecture of Ohsawa*

Abstract. We'll talk about some recent advances about optimal constant in L^2 extension and a conjecture of Ohsawa which is a high dimensional analogue of Suita conjecture.