Titles and Abstracts for The KSCV9 Symposium

16–20 July 2012

ABATE, Marco (Università di Pisa, Pisa, Italia)

Toeplitz operators and Carleson measures in strongly pseudoconvex domains

Abstract: We study mapping properties of Toeplitz operators associated to a finite positive Borel measure on a bounded strongly pseudoconvex domain $D \subset\subset \mathbb{C}^n$. In particular, we give sharp conditions on the measure ensuring that the associated Toeplitz operator maps the Bergman space $A^p(D)$ into $A^r(D)$ with $r > p$, generalizing and making more precise results by Čučković and McNeal [ČMc]. To do so, we give a geometric characterization of Carleson measures and of vanishing Carleson measures of weighted Bergman spaces in terms of the intrinsic Kobayashi geometry of the domain, along lines introduced in [AS] and generalizing to this setting results obtained by Kaptanoğlu [K] for the unit ball. (Joint work with J. Raissy and A. Saracco).

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BEDFORD, Eric (Indiana University, Bloomington, Indiana, U. S. A.)
Examples of rational mappings with interesting dynamics

Abstract: We will talk about some rational mappings in 2 and 3 dimensions. The 2-dimensional maps are related to maps discussed in Dynamics of rational surface automorphisms. Holomorphic dynamical systems, 57?104, Lecture Notes in Math., 1998, Springer, Berlin, 2010. The 3-dimensional maps are related to the pseudo-automorphisms of 3-space work, which we discuss in work with Kyounghee Kim: arXiv:1101.1614.

BRACCI, Filippo (Università di Roma "Tor Vergata, Roma Italia)
Loewner theory on complete hyperbolic manifolds

Abstract: The parametric method introduced by Ch. Loewner and implemented by P.P. Kufarev and Ch. Pommerenke for the study of increasing families of simply connected domains in the plane gave rise to many deep and important results (such as the proof of the Bieberbach conjecture) and got recently new fame thanks to the introduction of its stochastic version (SLEs) due to O. Schramm SLEs. In higher dimension, the Loewner theory has been extended mainly to the unit ball of $\mathbb{C}^n$ by, among others, J.A. Pfaltzgraff, I. Graham, G. Kohr, M. Kohr, H. Hamada.

In the recent years, the speaker together with his coauthors revised and extended the classical theory with a point of view based on hyperbolic metric and semigroups theory and showed how to extend completely the theory to complete hyperbolic manifolds. The accent is based on the so-called "evolution families" (families of semigroups-like holomorphic self-maps of the given manifold) and "Herglotz vector fields" (non-autonomous holomorphic vector fields which are semicomplete for almost all fixed time), and it is proved that there is a one-to-one correspondence between the two objects, given by a Loewner-type ODE. Loewner chains are then introduced in an abstract way as direct limit of the corresponding evolution families and they solve a Loewner-Kufarev type PDE. The problem of finding Loewner chains with appropriated image (the "Loewner range") starting from domains in $\mathbb{C}^n$ is related to resonances and to the Bedford conjecture..

In this talk I will explain the previous ideas.

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CHEN, Bo-Yong (Tongji University, Shanghai, China)
Weighted Bergman spaces and the $\bar{\partial}$ equation

Abstract. We study the weighted Bergman space $A^2_\alpha(\Omega)$ for any bounded pseudoconvex domain Ω with C^2 boundary. Our analysis is based on a new Hörmander type L^2-estimate for the d-bar equation with nonplurisubharmonic weights. Several applications including the corona problem, Levi problem and approximation problem in $A^2_\alpha(\Omega)$ are given. We also get a Gehring type upper estimate for functions in $A^2_\alpha(\Omega)$ and show that this estimate is optimal. A vanishing theorem of $A^2_\alpha(\Omega)$ is obtained for arbitrary bounded pseudoconvex domains (whose boundaries are not necessary C^2), together with an application to the upper bound of the order of hyperconvexity for hyperconvex domains.

CHOE, Boo Rim (Korea University, Seoul, Korea)
Linear sums of composition operators on the Fock-Sobolev spaces

Abstract: Linear sums of composition operators acting on the Fock-Sobolev spaces of several variables are studied. We show that such an operator is bounded only when all the composition operators in the combination are bounded individually. So, cancelation phenomenon is not possible on the Fock-Sobolev spaces, in contrast to what have been known on other well-known function spaces over the unit disk. We also show the analogues for compactness and the membership in the Schatten classes. In particular, compactness and the membership in some/all of the Schatten classes turn out to be the same.

References

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FORNÆSS, John Erik (Trondheim University, Norway)
Extensions of line bundles

Abstract:This is join work with Nessim Sibony and Erlend Wold. We discuss Hartogs extension of holomorphic line bundles on q-convex domains.

References

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GAUSSIER, Hervé (Université Joseph Fourier, Grenoble, France)
On the embeddability of almost complex manifolds

Abstract: The talk will consist of two distinct parts. In the fisrt one, we will prove that any J-pseudoconvex domain $D = \{\rho < 0\}$ in an almost complex manifold (M, J) is hyperbolic in the sense of Gromov. In the second one we will study the question of the embeddability of compact almost complex manifolds in projective spaces.

HAN, Chong-Kyu (Seoul National University, Seoul, Korea)

Generalized Frobenius theorem on involutivity and overdetermined PDE systems

Abstract: Given a Pfaffian system $\theta = (\theta^1, \dots, \theta^s)$ on a smooth (C^∞) manifold M^m, vanishing of the torsion ($d\theta \bmod \theta$) implies that M is locally foliated by integral manifolds of the maximal dimension: there exists s-parameter family of integral manifolds of dimension $m - s := p$. We discuss generalizations to the cases of s'-parameter family of integral manifolds of dimension p', where $0 \le s' < s$, $2 \le p' < p$. As applications we present results obtained by the time of conference including

1) Local geometry of Levi-forms associated with the minimality and the existence of complex submanifolds of generic CR manifolds.

2) Partial integrability of almost complex structures.

3) Holomorphicity of Forelli type and the inverse CR extension problem.

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HASLINGER, Friedrich (University of Vienna, Wien, Austria)
Compactness of the $\overline{\partial}$-Neumann operator.

Abstract: We consider the $\overline{\partial}$-Neumann operator

$$N : L^2_{(0,q)}(\Omega) \longrightarrow L^2_{(0,q)}(\Omega),$$

where $\Omega \subset \mathbb{C}^n$ is bounded pseudoconvex domain, and

$$N_\varphi : L^2_{(0,q)}(\Omega, e^{-\varphi}) \longrightarrow L^2_{(0,q)}(\Omega, e^{-\varphi}),$$

where $\Omega \subseteq \mathbb{C}^n$ is a pseudoconvex domain and φ is a plurisubharmonic weight function.

Using a general description of precompact subsets in L^2-spaces we obtain a characterization of compactness of the $\bar{\partial}$-Neumann operator, which can be applied to related questions about Schrödinger operators with magnetic field and Pauli and Dirac operators and to the complex Witten Laplacian. In this connection it is important to know whether the Fock space

$$\mathcal{A}^2(\mathbb{C}^n, e^{-\varphi}) = \{f : \mathbb{C}^n \to \mathbb{C} \text{ entire } : \int_{\mathbb{C}^n} |f|^2 e^{-\varphi}\, d\lambda < \infty\}$$

is infinite-dimensional, which depends on the behavior at inifinity of the eigenvalues of the Levi matrix of the weight function φ.

In addition we discuss obstructions to compactness of the $\bar{\partial}$-Neumann operator.

References

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HWANG, Jun-Muk (Korea Institute for Advanced Study, Seoul, Korea
Holomorphic symplectic geometry

Abstract: A holomorphic 2-form ω on a $2n$-dimensional complex manifold is called a symplectic form, if $d\omega = 0$ and ω^n is a nowhere-vanishing $2n$-form. A complex manifold M equipped with a holomorphic symplectic form is called a holomorphic symplectic manifold. Holomorphic symplectic geometry studies the geometry and analysis of holomorphic symplectic manifolds, up to symplecto-biholomorphism. In other words, two holomorphic symplectic manifolds (M, ω) and (M', ω') are equivalent if there exists a biholomorphic map $\varphi : M \to M'$ with $\varphi^*\omega' = \omega$.

Holomorphic symplectic geometry is a natural holomorphic analogue of (the standard, C^∞-)symplectic geometry. The latter has its origin in mechanics and optics (e.g. [GS]). It is currently a very active area of research in differential geometry with a rapidly growing list of intricate and deep results. Many concepts and questions of symplectic geometry have counterparts in holomorphic symplectic geometry. But many of the results in symplectic geometry do not admit direct translation into holomorphic symplectic geometry.

Historically, holomorphic symplectic geometry has been studied mostly by algebraic geometers, as it naturally arises in the study of hyperkähler manifolds (see [Hu]). Consequently, most of the problems that have been extensively studied are related to algebraic geometry (see [Be]). However, the subject naturally belongs to complex geometry/several complex variables and many interesting problems arise from this point of view. I will give an introductory survey of these problems.

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ISHI, Hideyuki (Nagoya University, Nagoya, Japan

On estimates of the Bergman kernel of a homogeneous bounded domain

Abstract: Every homogeneous Siegel domain is realized as a set of complex matrices with specific block decompositions ([I,Xu]). Based on this fact, we develop a new method for the study of the Bergman kernels of homogeneous bounded domains and homogeneous Siegel domains ([I-Y]). As an application of the method, we discuss estimates of the length of the gradient of the logarithm of the Bergman kernel, which was studied in [D].

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JOO, Jae-Cheon (Pusan National University, Busan, Korea)
On the Uniqueness of CR mappings

Abstract: In this talk, I first prove the following theorem.

Theorem ([2]) *Let M be a smooth real hypersurface in $\mathbb{C}^n$ which does not contain any analytic hypersurface and let $p \in M$ be a strongly pseudoconvex point. Let $f : M \to M$ be a CR mapping. If $f(p) = p$, $df_p = \mathrm{Id}$ and the iteration $\{f^k : k \geq 0\}$ is relatively compact in the compact-open topology of M, then $f \equiv \mathrm{Id}$.*

This theorem can be regarded as a CR version of the Cartan Uniqueness Theorem for holomorphic mappings. I also would like to introduce a uniqueness theorem by J. Byun, M. Song and myself ([1]) for holomorphic vector fields tangent to a real hypersurface singular at an infinite type point.

References

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KAI, Chifune (Kanazawa University, Kanazawa, Japan)
The representative domain of a homogeneous bounded domain

Abstract: The representative domain introduced by S. Bergman gives a nice realization for a homogeneous bounded domain, which is a generalization of the Harish-Chandra realization for a symmetric bounded domain. We show that the representative domain coincides with the image of the Cayley transform introduced by R. Penney and T. Nomura. As an application, we see that a homogeneous bounded domain is symmetric if and only if its representative domain is convex.

This talk is based on the joint paper with H. Ishi [2].

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KIM, Dano (Seoul National University, Seoul, Korea)
Remarks on Extension and Non-vanishing of sections

Abstract: L^2 extension of Ohsawa-Takegoshi type for an irreducible subvariety Y in a complex analytic variety X is especially well known for the case when X and Y are nonsingular and Y is of codimension 1. We will first discuss more general cases of extension with motivations coming from algebraic geometry. One can even consider the case where Y is not necessarily irreducible, as was recently considered by M. Paun in the context of his recent work with Demailly and Hacon. All these general cases are relevant to the non-vanishing problem in the minimal model program of algebraic geometry. We will talk about extension results with Y being a log-canonical center and about how the notion of a log-canonical center could possibly be generalized in view of the above problem. We will also discuss related topics such as the asymptotic behavior of Bergman kernels as in the works of Berndtsson and Popovici.

LE Hai Khoi (Nanyang Technological University (NTU), Singapore)

Mutual dualities between $A^{-\infty}(\Omega)$ and $A^{\infty}(\widetilde{\Omega})$ for lineally convex domains

Abstract: Let Ω be a bounded domain in $\mathbb{C}^n$ $(n \geq 2)$. Without loss of generality, we can assume that $0 \in \Omega$. Put $d(z) = d_\Omega(z) := \inf\limits_{w \in \partial\Omega} |z - w|,\ z \in \Omega$. The space $A^{-\infty}(\Omega)$ is defined as follows:

$$A^{-\infty}(\Omega) := \left\{ f \in \mathcal{O}(\Omega) : \exists k \in \mathbb{N}, C > 0 \text{ such that } \sup_{z \in \Omega} |f(z)|\ [d(z)]^k \leq C \right\}.$$

Notice that the condition in the definition of $A^{-\infty}(\Omega)$ is the familiar polynomial growth condition $\sup\limits_{z \in \Omega}(1 - |z|)^k |f(z)| \leq C$ if the domain Ω is the open unit ball.

The space $A^{-\infty}(\Omega)$ can be equipped with its natural inductive limit topology of Banach spaces

$$A^{-k}(\Omega) := \left\{ f \in \mathcal{O}(\otimes) : \|\{\|_\| = \sup_{\ddagger \in \otimes} |\{(\ddagger)|\ [\lceil(\ddagger)]^\| < +\infty \right\},\ k = 1, 2, \ldots,$$

and hence becomes a (DFS)-space.

We also consider the (FS)-space $A^{\infty}(\widetilde{\Omega})$ of all holomorphic functions in $int\ \widetilde{\Omega}$, the interior of $\widetilde{\Omega}$, which are in $C^{\infty}(\widetilde{\Omega})$; i.e.,

$$A^{\infty}(\widetilde{\Omega}) := \mathcal{O}\big(int\ \widetilde{\Omega}\big) \cap C^{\infty}(\widetilde{\Omega}),$$

endowed with the topology given by the system of norms $|f|_k = \sup\limits_{\|\alpha\| \leq k} \sup\limits_{\zeta \in \widetilde{\Omega}} |D^\alpha f(\zeta)|,\ k = 0, 1, 2, \ldots$. Here $\widetilde{\Omega}$ is the conjugate set of Ω; i.e., $\widetilde{E} := \{\zeta \in \mathbb{C}^n : \ \langle z, \zeta \rangle \neq 1 \ \text{ for any } z \in E\}$.

In our recent papers [1,2], for a bounded convex domain Ω of $\mathbb{C}^n$, we established, via the Laplace transformation, the mutual dualities between $A^{-\infty}(\Omega)$ and the space $A_\Omega^{-\infty}$ of entire functions in $\mathbb{C}^n$ with a certain growth condition. We also gave an explicit construction of a countable sufficient set for the dual space and then, applying the so-called "dual relationship" we showed that any function from either $A^{-\infty}(\Omega)$ or $A_\Omega^{-\infty}$ can always be represented in the form of a Dirichlet series.

A natural question to ask is: how about more general, say lineally convex, domains? In the present talk we give a solution to this problem.

The results are based on joint work with A.V. Abanin.

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LEE, Kang-Hyurk (GyeongSang National University, Jinju, Korea)
Problems on the bounded realization of unbounded model domains

Abstract: In Several Complex Variables, it has been a fundamental problem to classify domains which can play the same role of model objects as the unit disc. Many theories (for instance, [7,1]) said that most of model domains can be realized as certain unbounded domain, so called unbounded model in this talk. Although these model domains are unbounded, they have many affine automorphisms; hence this representation of models has been employed in wide area of Several Complex Variables.

In this talk, I will introduce briefly theories on this topic and then discuss problems on the realization of an unbounded model as a bounded domain. Especially I will mention my current collaboration [2] with Jisoo Byun on the smoothly bounded realization of unbounded models with real-analytic boundary. See [1, 5, 6].

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OHSAWA, Takeo (Nagoya University, Nagoya, Japan)
On the normal bundles of Levi flats in Kähler manifolds

Abstract: Combining the L^2 method of solving the $\overline{\partial}$ equation with a method of Brunella in the study of Levi flats as stable sets of holomorphic foliations, two nonexistence theorems are proved for Levi flats in compact Kähler manifolds satisfying certain semipositivity conditions on the normal bundles.

POLETSKY, Evegeny (Syracuse University, Syracuse, New York, U. S. A.)
Holomorphic homotopy theory

Abstract: Holomorphic homotopy theory studies continuous deformations of holomorphic mappings and the major question is when one holomorphic mapping can be continuously deformed into another holomorphic mapping via holomorphic mappings. We call such mappings h-homotopic.

The serious studies of such questions was initiated by M. Gromov in [3] who was interested in the homotopical Oka principle: when homotopic holomorphic mappings are h-homotopic? It led to the notions of Oka and elliptic manifolds and many interesting applications.

Later this theory obtained was greatly advanced by F. Forstnerič. In [1] and [2] one can find an excellent discussion of arising problems and there solutions in many cases.

Recently h-homotopical constructions appeared on non-elliptic manifolds which are much more general. For example, in the description of B. Jöricke[4] of envelopes of holomorphy and the disk formula for plurisubharmonic subextensions by F. Larusson and the speaker. These results raised an interest to h-homotopies on general complex manifolds.

In the talk we will briefly present Gromov's theory and then discuss the h-homotopy theory for general manifolds including the results of Jöricke and Larusson-Poletsky. Finally, we will show how an h-analog for the fundamental group can be introduced.

References

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SCHMALZ, Gerd (University of New England, Armidale, Australia)
Classification of rigid spheres

Abstract: In 1991 N. Stanton gave a list of rigid spheres, that is rigid hypersurfaces in $\mathbb{C}^2$ that are given in normal form and are equivalent to the Heisenberg sphere. She asked the question whether this list was complete. I will give a report on joint work in progress with Vladimir Ezhov on finding a complete classification of such rigid spheres.

References

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SHCHERBINA, Nikolay (University of Wuppertal, Wuppertal, Germany)
On defining functions for unbounded domains

Abstract: The existence of strictly plurisubharmonic defining functions for bounded strongly pseudoconvex domains is an old result which has many applications in complex analysis. Simple examples show that in the case of unbounded domains strictly plurisubharmonic defining functions might not exist. That is why in the unbounded case we need to weaken this notion. Namely, instead of strictly plurisubharmonic functions we consider plurisubharmonic functions which are strictly plurisubharmonic only in a neighbourhood of the boundary. For this class of defining functions we prove the following general existence result.

Theorem. *Let X be a paracompact complex space and let $\Omega \subset X$ be a strongly q-pseudoconvex domain. Then there exists a $\mathcal{C}^2$-smooth q-plurisubharmonic function φ defined on a neighbourhood of $\overline{\Omega}$ such that $\Omega = \{\varphi < 0\}$ and φ is strictly q-plurisubharmonic near $b\Omega$.*

In the case when X is a complex manifold and $\Omega \subset X$ is a strongly pseudoconvex domain we also define the *kernel* $K(\Omega)$ of Ω as the set of all points $z \in \Omega$ such that every defining function for Ω fails to be strictly plurisubharmonic at z. We discuss then the *Wermer type* sets (for more details see [HST]) and prove for them Liouville type theorem. Using it we show that there is an unbounded strongly pseudoconvex domain in $\mathcal{C}^2$ such that its kernel coinsides with the Wermer type set (and hence has no analytic structure).

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DO Duc Thai (Hanoi National University of Education, Hanoi, Vietnam)
Holomorphic mappings into compact complex manifolds

Abstract: The purpose of this talk is to present a second main theorem with the explicit truncation level for holomorphic mappings of $\mathbb{C}$ (or of a compact Riemann surface) into a compact complex manifold sharing divisors in subgeneral position. Finally, we will give some applications of the above main theorems. Namely, we show a unicity theorem for holomorphic curves of a compact Riemann surface into a compact complex manifold sharing divisors in N-subgeneral position. Moreover, we also generalize the Five-Point Theorem of Lappan to a normal family from an arbitrary hyperbolic complex manifold to a compact complex manifold. Here is a work joining with Vu Duc Viet.

WULCAN, Elizabeth (Chalmers Institute of Technology, Göteborg, Sweden)
Realizing Serre duality as a product of currents

Abstract: I will discuss a joint work in progress with Hakan Samuelsson Kalm and Jean Ruppenthal. Given an analytic space X, we define (by modifying a recent construction by Andersson-Samuelsson Kalm) fine sheaves $\mathcal{A}_X^{0,q}$ of $(0,q)$-currents that are smooth on the regular part of X and that give a resolution of the structure sheaf $\mathcal{O}_X$. If X is Gorenstein we also introduce sheaves $\mathcal{A}_X^{n,q}$ of (n,q)-currents that give a resolution of the dualizing sheaf ω_X. The sheaf $\mathcal{A}_X^{*,*}$ has a multiplicative structure; there is a well-defined wedge product $\mathcal{A}_X^{p,q} \times \mathcal{A}_X^{p',q'} \to \mathcal{A}_X^{p+p',q+q'}$, which gives a pairing $H^{n-q}(X,\mathcal{O}_X) \times H^q(X,\omega_X) \to \mathbb{C}$ by $([\alpha],[\beta]) \mapsto \int \alpha \wedge \beta$. This pairing is non-degenerate and thus realizes Serre duality on X.

ZAITSEV, Dmitri (Trinity College Dublin, Dublin, Ireland)
Normal forms for non-integrable almost CR structures

Abstract: Constructions extending the Chern-Moser normal forms to non-integrable Levi- nondegenerate (hypersurface type) almost CR structures. One of them translates the Chern-Moser normalization in the pure intrinsic setting, whereas the other directly extends the (extrinsic) Chern-Moser normal form by allowing non-CR embeddings that are in some sense "maximally CR". One of the main differences with the classical integrable case is the presence of the non-integrability tensor at the same order as the Levi form, making impossible a good quadric approximation—a key tool in the Chern-Moser theory. Partial normal forms are obtained for general almost CR structures of any CR codimension, in particular, for almost-complex structures. Applications are given to the equivalence problem and the Lie group structure of the group of all CR-diffeomorphisms. These normal forms require an additional nondegeneracy condition called "strong nondegeneracy" involving the non-integrability tensor. In a further unified normal form we are able to remove that condition by modifying the normalization condition involving higher order terms than the ones used by Chern and Moser.

References

[CM74] Chern, S.S; Moser, J.K.: Real hypersurfaces in complex manifolds. Acta Math. 133 (1974), 219–271.

[Z12] Zaitsev, D.: Normal forms for nonintegrable almost CR structures. Amer. J. Math. 134 (2012), 1–33; http://arxiv.org/abs/0902.2687

ZHANG, Liyou (Capital Normal University, Beijing, China)
Squeezing functions and geometry of bounded domains

Abstract: We introduce the notion of squeezing functions on bounded domains in $\mathbb{C}^n$ and study some properties of them. The so-called *holomor- phic homogeneous regular domains* introduced by Liu-Sun-Yau and the *uniformly squeezing domains* introduced by S.-K. Yeung are exactly the domains whose squeezing functions have positive lower bounds. In dimension one case, we get a neat description of boundary behavior of squeezing functions of finitely connected planar domains. This gives a necessary and sufficient condition for a finitely connected planar domain to be a holomorphic homogeneous regular domain. Consequently, we can recover some important results in complex analysis. For annuli, we obtain some interesting properties of their squeezing functions. We also consider the limit of squeezing functions of a sequence of bounded domains and give some comparisons of intrinsic positive forms and metrics in terms of squeezing functions. After introducing the notions of (in- trinsic) ball pinching radius, we estimate the bounded behavior of squeezing functions by using these data. Applying this conclusion, it turns out that all Cartan-Hartogs domains are homogenous-regular which gives a positive answer to a question by Yin in 2009. Finally, we prove all convex domains are also homogenous regular and consider the bounded estimates of squeezing functions for strongly convex domains. The talk is based on joint works with Q. Guan, F. Deng and K.-T Kim.

ZHOU, Xiang-Yu (Chinese Academy of Science, Beijing, China)
Optimal constant problem in L^2 extension theorem

Abstract: We will talk about some recent results obtained in the following articles and some preprints on two different problems: (1) optimal constant problem in L^2 extension theorem and, (2) rigidity problem of automorphism groups of invariant domains in Stein homogeneous spaces.

References

1. Q.A. Guan, X.Y. Zhou, and L.F. Zhu, On the Ohsawa-Takegoshi L^2 extension theorem and the twisted Bochner-Kodaira identity. C. R. Acad. Sci. Paris 349(2011), 797-800.

2. Q.A. Guan, X.Y. Zhou, and L.F. Zhu, L^2 extension theorem and the Bochner-Kodaira identity with non-smooth twist factor. Liouville's J. online, 2012.

3. F.S. Deng and X.Y. Zhou, Rigidity of automorphism groups of invariant domains in certain Stein homogeneous manifolds. C. R. Acad. Sci. Paris, online, 2012.

ZHU, Kehe (State University of New York, Albany, New York, U. S. A.)
Intricacies of Fock Spaces

Abstract: For $0 < p \leq \infty$ and $\alpha > 0$ let L^p_α denote the space of Lebesgue measurable functions $f(z)$ on the complex plane C such that the function $f(z)e^{-\alpha|z|^2/2}$ belongs to $L^p(C, dA)$, where dA is ordinary area measure. The Fock space F^p_α is then just the subspace of L^p_α consisting of entire functions.

The definition of Fock spaces is similar to that of the more well-known Hardy and Bergman spaces. In fact, the three families of spaces share many properties and the study of them often follows the same theme. The purpose of this talk is to publicize several interesting results that are unique for the Fock space setting.

More specifically, I will talk about the Banach duality, complex interpolation, and zero sets for Fock spaces. There are striking differences between the results for Fock spaces and the corresponding ones for Hardy and Bergman spaces. I will also talk about some properties of the Berezin transform that are special for Fock spaces.

The main references for the talk are the following:

[1] S. Janson, J. Peetre, and R. Rochberg, Hankel forms and the Fock space, Revista Mat. Ibero-Amer. 3 (1987), 61-138.

[2] K. Zhu, Analysis on Fock Spaces, Springer, 2012.