Titles and Abstracts of

The GAIA-PNU Workshop 2013-2 on Complex Analysis and Geometry

Speaker: Hong Rae Cho (Pusan National University)

Title: Hermite-Sobolev and Fock-Sobolev spaces

Abstract: Let $s \geq 0$. The Fock-Sobolev spaces $F_R^{s,2}$ of fractional order s are introduced through the fractional radial derivatives R^s. We show that the Bargmann transform is a unitary isomorphism between the Hermite-Sobolev spaces $W^{s,2}(\mathbb{R}^n)$ with the Fock-Sobolev spaces $F_R^{s,2}$. Moreover, we prove that the Fock-Sobolev spaces $F_R^{s,2}$ are identified with the weighted Fock spaces F_s^2 that do not involve derivatives. So, the study on the Fock-Sobolev spaces is reduced to that on the weighted Fock spaces.

References

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Speaker: Xianghong Gong (U of Wisconsin and GAIA of POSTECH)

Title: Regularity in the local CR embedding problem

Abstract : We consider a formally integrable, strictly pseudoconvex CR manifold M of hypersurface type of dimension at least 7. It is known by works of Kuranishi and Akahori that locally, M can be holomorphically embedded into $\mathbf{C}^n$ when the structure is smooth. We show that if the CR structure of M is of class C^m for an integer m with $4 \leq m \leq \infty$, there is a local CR embedding that is of class C^a for every $a < m + 1/2$. This is a joint work with S. M. Webster.

Speaker: Su Kyung Han (Graduate student, Pusan National University)

Title: Peak functions for Bergman spaces with exponential type weight on the unit disc

Abstract: Let $A^p_{\alpha,\beta}$ be the Bergman space with the weight function $\omega_{\alpha,\beta}(z) = (1-|z|)^\alpha e^{-\frac{\beta}{1-|z|}}$, where $\alpha \in \mathbb{R}$ and $\beta > 0$. It is known that having an appropriate peak function in $A^p_{\alpha,\beta}$ can be useful toward a characterization of the Carleson measures for $A^p_{\alpha,\beta}$. We introduce a simple explicit formula of the peak function for $A^p_{\alpha,\beta}$.

References

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Speaker: Seungro Joo (Graduate student, POSTECH)

Title: On the Characterization of models in $\mathbb{C}^2$

Abstract: Classification of domains with a noncompact automorphism groups is a well-known problem. The Wong-Rosay theorem yields this for strongly pseudoconvex domains. The case of weakly pseudoconvex domains is still not completely understood. In this talk, we present the following result in $\mathbb{C}^2$ and its proof, given by Berteloot.

Let Ω be a domain in $\mathbb{C}^2$. Assume that there exist a point z_0 in Ω and a sequence $\{\phi_p(z_0)\}$ of holomorphic automorphisms of Ω such that $\{\phi_p(z_0)\}$ is accumulating at some point ζ_0 where $\partial\Omega$ is pseudoconvex and finite type near ζ_0. Then Ω is biholomorphically equivalent to its model at ζ_0.

References

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Speaker: Sejun Kim (Graduate student, POSTECH)

Title: On the Joo-Kim-Schmalz generalization of Forelli's theorem

Abstract: Classical Forelli's theorem states:

Theorem (Forelli 1977)**.** *If a function $f : \mathbb{B}^n \to \mathbb{C}$ satisfies the following two two conditions:*

(*i*) $f \in C^\infty(0)$,
(*ii*) *for every unit vector $v = (v_1, \ldots, v_n) \in \mathbb{C}$, $f(\zeta v_1, \ldots, \zeta v_n)$ is holomorphic in the single complex variable ζ with $|\zeta| < 1$,*

then f is holomorphic.

Its generalization started in 2005 by E. M. Chirka in complex dimension 2, with an open-end question whether the generalization holds in higher dimensions. The problem was solved by Joo-Kim-Schmalz in [3], and we would like to present a sketch of their proof in this talk.

References

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Speaker: Han-Wool Lee (Graduate student, Pusan National University)

Title: Sobolev version of some generalized Fock space

Abstract: In Cho-Zhu, classical Fock-Sobolev spaces are characterized by weighted Fock spaces. In this talk, we would like to introduce a generalization of that result. In fact, we prove that $f \in F^{p,m}_\alpha$ if and only if $z^{m(\alpha-1)} f \in F^p_\alpha$, where $F^p_\alpha = \{f \in H(\mathbb{C})\} : \int_{\mathbb{C}} |f(z)e^{-\frac{1}{2}|z|^\alpha}|^p dA(z) < \infty\}$ and $F^{p,m}_\alpha = \{f \in F^p_\alpha : \sum_k^m \|\partial^k f\|^p_{F^p_\alpha} < \infty\}$.

References

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Speaker: Takeo Ohsawa (Nagoya University)

Title: Application and simplification of the L^2 extension theorem

Abstract: As an application of a sharp L^2 extension theorem for holomorphic functions obtained in the works of Błocki and Guan-Zhou, a stability theorem for the boundary asymptotics of the Bergman kernel is proved. An alternate proof of the extension theorem is given. It is a simplified proof in the sense that it is free from ODE.

Speaker: Atsushi Yamamori (Researcher, GAIA of POSTECH)

Title: On origin-preserving automorphisms and the Bergman mapping

Abstract: In this talk, we study origin-preserving holomorphic automorphisms of quasi-circular (m-circular) domains. We introduce the normality for the quasi-circular domains and prove that every origin-preserving automorphisms of the normal quasi-circular domains is linear. This talk is based on the following papers.

References

[1] H. Kim, V. T. Ninh and A. Yamamori, The automorphism group of a certain unbounded non-hyperbolic domain, J. Math. Anal. Appl. 409 (2014), 637–642.
[2] A. Yamamori, Automorphisms of normal quasi-circular domains, Bull. Sci. Math., to appear.

Speaker: Jongho Yang (Graduate student, Korea University)

Title: Commuting Toeplitz operators on the Fock-Sobolev space

Abstract: We study the commuting condition of two Toeplitz operators acting on the Fock-Sobolev spaces over the complex plane. With the symbol class satisfying a certain growth condition at infinity, if we further assume that one symbol is nonconstant and radial, then we show that the other must also be radial.

References

[1] W. Bauer and Y. J. Lee, *Commuting Toeplitz operators on the Segal-Bargmann space*, J. Funct. Anal. **260** (2011), no.2, 460–489.
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Speaker: Jihun Yum (Graduate student, POSTECH)

Title: Characterization of Complex Manifolds by the Isotropy Subgroups of Their Automorphism Groups

Abstract: In [1], the following remarkable theorem has been given:

Theorem (Greene-Krantz)**.** *Let M be a noncompact complex manifold of complex dimension n. Let $p \in M$. Assume that there is a compact subgroup H of the isotropy subgroup I_p at p, of the holomorphic automorphism group of M, with the following property:*

> *for any two real tangent vectors η, ζ at p there is an element $h \in H$ such that $dh|_p(\eta) = \lambda\zeta$ for some real number λ.*

Then M is either biholomorphic to the unit ball in $\mathbb{C}^n$ or biholomorphic to $\mathbb{C}^n$.

We shall present the main theorem, its ideas, and a sketch of the proof, following [1] and [2].

References

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